How to Design Mathematics Lessons based on the Realistic Approach?

by: Zulkardi

1. Introduction

This report describes and synthesizes literature that was surveyed on the topic *realistic mathematics education (RME)*. This topic will be part of the literature review chapter of the Ph.D. research proposal. The title of the research is Computer Assisted Curriculum Analysis, Design and Evaluation for Mathematics Education in Indonesia (CASCADE-MEI). The main aim of this study is to develop a computer-support system which supports students teachers in Indonesia designing mathematics lesson plans based on the realistic approach.

Problem Definition

The literature review started with the following problem definition: 'how to design mathematics lesson plans based on the realistic approach? This problem definition was split up into several smaller units of questions that focus on one aspect of the problem definition. It was hoped for that the survey would become more manageable. These smaller units have been formulated as questions. The answers to these questions constitute the sections that make up this report. The questions were:

- *What is realistic mathematics education?*
- *What are the characteristics of realistic mathematics education?*
- *Howis the realistic approach related to the constructivist approach?*
- *Howto design mathematics lessons based on the realistic approach?*

Search Method

The search began with several talks with the mentor and an expert on realistic mathematics education. From all of them some books were borrowed. They gave several hints in which directions to search such as the type of report and the recently year of publications. With the problem definition in mind a list of possible key words was made. Literature has been sought with the following key phrases: realistic mathematics education, assessment methods in mathematics education, lesson development, and constructivism in mathematics education. The last keyword is used in order to compare it to RME approach.

Two ways of searching were used in this review. The first way of searching was through the internet with all of the above mentioned keywords. The goal of this search was to find recent publications such as journals and web sites. By using some search engines such as yahoo (at address http://www.yahoo.com) and altavista (at address http://www.altavista.com) the exact phrase of keywords were typed in advanced search part and the results appeared. They were web-sites and web-pages that contain on-line articles. The first and the most useful keyword is realistic mathematics education, because some of the results (14 web sites by Yahoo or 24 web sites by Alta vista are) deal with RME. The results from others key words are assessment in mathematics education(86 web sites), lesson development (363 web sites) and constructivism in mathematics education (53 web sites). However, most of the results were found only the list of references, instead of the articles. The most useful web sites that are selected that contains online articles and resources are:

- Freudenthal Institute (address at http://www.fi.ruu.nl), contains some on-line articles about realistic mathematics education such as the remesa project and norma project.

- NCTM organization (address http://www.nctm.org) that contains online journal and teaching standards of mathematics in USA, and
- The Third International Mathematics and Science Study (TIMSS) is the largest and most ambitious international study of student achievement ever conducted. The address of TIMSS center is http://wwwcsteep.bc.edu/timss

- A number of on-line articles about the constructivist approach also available in a web-page t the following address:http://www.inform.umd.edu:8080/UMS+State/UMD-Projects/ MCTP/ W W W / Essays.html.
- A lesson development page http://www.eirc.org/integrate/tsld015.htm that contains steps in developing lesson plan and an example of lesson development software at http://www.deixissoftware.com/

Another way of searching was a search through libraries. Three different libraries were searched, SLO library in Enschede, TO (the automated online catalog of the library of the Faculty of Educational Science and Technology) University of Twente, and central library of the University of Twente. Some articles in the Journal of Research in Mathematics Education and the Journal of Educational Studies in Mathematics found in at SLO library in Enschede. In the TO library, by using on-line catalog, eight of books about realistic mathematics and also some journals were found. While in the central library of University of Twente, a dissertation about realistic mathematics was found. In addition, two books used during the realistic mathematics course were very useful.

2. What is Realistic Mathematics Education?

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education that was first introduced and developed by the Freudenthal Institute in the Netherlands. This theory has been adopted by a large number of countries all over the world such as England, Germany, Denmark, Spain, Portugal, South Africa, Brazil, USA, Japan, and Malaysia (de Lange, 1996).

The present form of RME is mostly determined by Freudenthal's view on mathematics (Freudenthal, 1991). Two of his important points of views are mathematics must be connected to reality and mathematics as human activity. First, mathematics must be close to children and be relevant to every day life situations. However, the word ‘realistic’, refers not just to the connection with the real-world, but also refers to problem situations which real in students' mind. For the problems to be presented to the students this means that the context can be a real-world but this is not always necessary. De Lange (1996) stated that problem situations can also be seen as applications or modeling.

Second, the idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning processes while the meaning of guided is the instructional environment of the learning process. For example, the history of mathematics can be used as a source of inspiration for course design. Moreover, the reinvention principle can also be inspired by informal solution procedures. Informal strategies of students can often be interpreted as anticipating more formal procedures. In this case, the reinvention process uses concepts of mathematization as a guide.

Two types of mathematization which were formulated explicitly in an educational context by Treffers (1987) are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. The following activities are examples of horizontal mathematization: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical problem. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. The following activities are example of vertical mathematization: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing.

Freudenthal (1991) stated that "horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols." But he adds that the difference between these two types is not always clear cut.

Figure 1 illustrates the process of reinvention. It shows that both the horizontal and vertical mathematization take place in order to develop basic concepts of mathematics or formal mathematical language.

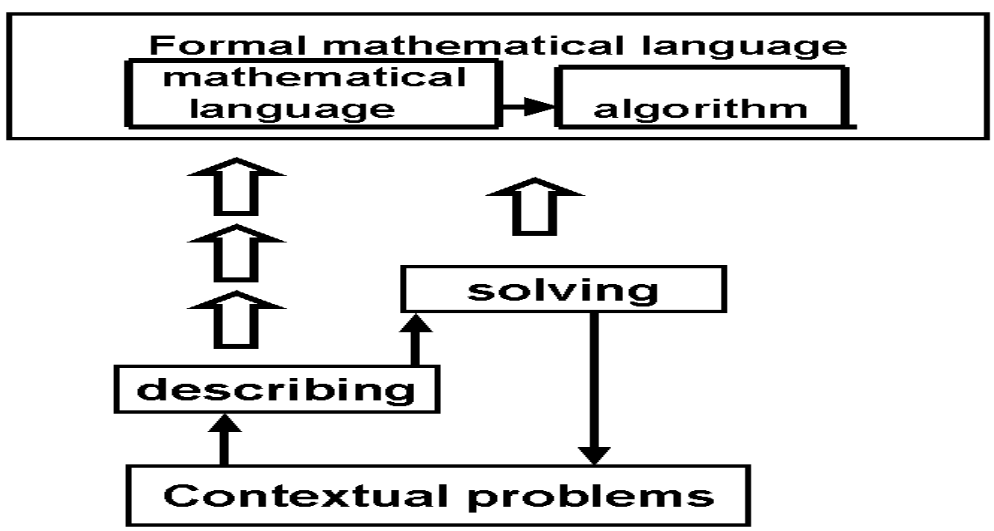


Figure 1. Guided Reinvention model (Gravenmeijer, 1994)

The learning process starts from contextual problems. Using activities in the horizontal mathematization, for instance, the student gains an informal or a formal mathematical model. By implementing activities such as solving, comparing and discussing, the student deals with vertical mathematization and ends up with the mathematical solution. Then, the student interprets the solution as well as the strategy which was used to another contextual problem. Finally, after the student has used the mathematical knowledge.

Treffers classifies mathematics education into four types with regard to horizontal and vertical mathematization(see table 1). These classifications are described clearly by Freudenthal (1991):

- *mechanistic*, or 'traditional approach', is based on drill-practice and patterns, which treat the person like a computer or a machine(mechanic). It means the activities of students in this approach are based on memorizing a pattern or an algorithm. The errors will be occurred if the students are faced with other problems that are different from the one they have memorized. In this approach, both horizontal and vertical mathematization are not used.
- *Empiristic approach*, the world is a reality, in which students are provided with materials from their living world. This means students are faced with the situations in which they have to do horizontal mathematization activities. However, they are not prompted to the extended situation in order to come up with a formula or a model. Treffers (1991) pointed out that this approach, in general, it is one that is not taught .
- *structuralist*, or 'New Math approach' that is based on set theory, flowchart and games that are kinds of horizontal mathematization but they are stated from an 'ad hoc' created world, which had nothing in common with the learner's living world.
- *realistic approach*, a real-world situation or a context problem is taken as the starting point of learning mathematics. And then it is explored by horizontal mathematization activities. This means students organize the problem, try to identify the mathematical aspects of the problem, and discover regularities and relations. Then, by using vertical mathematization students develop mathematical concepts.

Table 1: Four types of mathematics education (Freudenthal,1991)

Type	**Horizontal** **Mathematization**	**Vertical** **mathematization**
Mechanistic	-	-
Empiristic	+	-
Structuralist	-	+
Realistic	+	+

In literature several positive results of the RME theory can be found. For instance, in the USA, RME is adopted in the "*Mathematics in context*" textbooks for grade 5-8. After the books were used by students in several school districts from different states, a preliminary research showed that the students' achievement on the national test highly increased (Romberg & de Lange, 1998). Furthermore, in the country where RME originally has been developed, the Netherlands, there are also positive results that can be used as indicators for the success of RME in the reform of mathematics education. The results of the Third International Mathematics and Science Study (TIMSS) show that students in the Netherlands gained high achievements in mathematics education (Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1997).

3. What are the characteristics of RME?

Historically, the characteristics of RME is related to the Van Hiele's levels of learning mathematics. According to Van Hiele (cited in de Lange, 1996) the process of learning proceeds through three levels: (1) a pupil reaches the first level of thinking as soon as he can manipulate the known characteristics of a pattern that is familiar to him/her; (2) as soon as he/she learns to manipulate the interrelatedness of the characteristics he/she will have reached the second level; (3) he/she will reach the third level of thinking when he/she starts manipulating the intrinsic characteristics of relations.

Traditional instruction is inclined to start at the second or third level, while the realistic approach starts from first level. Then, in order to start at the first level that deals with the phenomenon that is familiar to the students, Freudenthal's didactical phenomenology that learning should start from a contextual problem, is used. Furthermore, by guided reinvention and progressive mathematizations, students are guided didactically to process as efficiently from one level to another level of thinking through mathematization.

The combinations of the three Van Hiele's levels, Freudenthal's didactical phenomenology and Treffers' progressive mathematization result in the following five basic characteristics of realistic mathematics education:

- *phenomenological exploration or the use of contexts;*
- *the use of models or bridging by vertical instruments;*
- *the use of students own productions and constructions or students contribution;*
- *the interactive character of the teaching process or interactivity; and*
- *the intertwining of various learning strands.*

These characteristics will be elaborated in the next paragraphs.

(1) Phenomenological exploration or the use of contexts

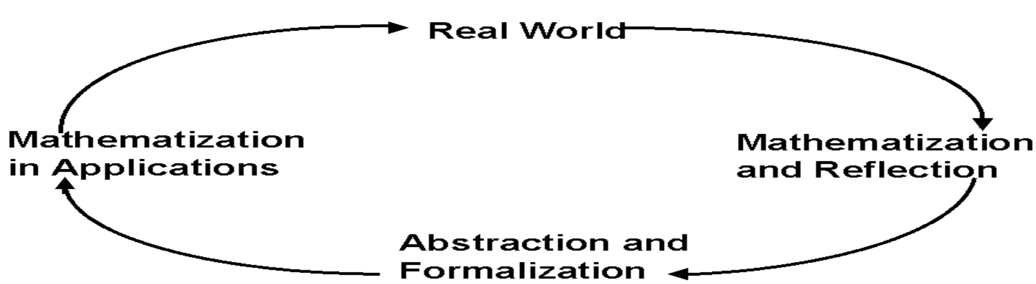


Figure 2. Concept and applied Mathematization (De Lange, 1996)

In RME, the starting point of instructional experiences should be `real' to the students; allowing them to immediately become engaged in the situation. This means that instruction should not st*art with the formal system. The phenomena by which the concepts appear in reality* should be the source of concept formation. The process of extracting the appropriate concept from a concrete situation is stated by De Lange (1987) as 'conceptual mathematization'. This process will force the students to explore the situation, find and identify the relevant mathematics, schematize, and visualize to discover regularities, and develop a 'model' resulting in a mathematical concept. By reflecting and generalizing the students will develop a more complete concept. Then, the students can and will apply mathematical concepts to new areas of the real world and by doing so, reinforce and strengthen the concept. This process is called applied mathematization(figure 2).

(2) The use of models or bridging by vertical instruments

The term model refers to situation models and mathematical models that are developed by the students themselves. This means that the students develop models in solving problems. At first, the model is a *model of* a situation that is familiar to the students. By a process of generalizing and formalizing, the model eventually becomes an entity on its own. It becomes possible that is used as a *model for* mathematical reasoning. Four levels of models in designing RME lessons is described below (figure 3):

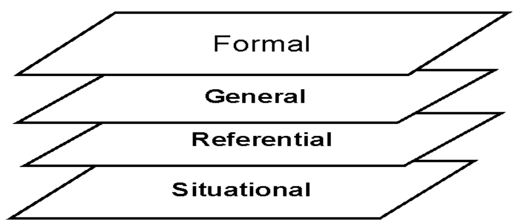


Figure 3. Levels of models in RME (Gravenmejer, 1994)

- *the situational level*, where domain-specific, situational knowledge and strategies are used within the context of the situation;
- *a referential level or the level 'model of'*, where models and strategies refer to the situation described in the problem;
- *a general level or the level 'model for',* where a mathematical focus on strategies dominates over the reference to the context; and
- *the level of formal mathematics,* where one works with conventional procedures and notations.

An example of a lesson using these four models is long division (Gravenmeijer, 1994).

In the first level, long division is associated with real-life activities. For example, sharing sweets among the children. Here, the students bring in their situational knowledge and strategies and apply them in the situation. The second level is entered when the same sweets division is presented as a written task and the division is modeled with paper and pencil. Then, the focus is shifted towards strategies from a mathematical point of view. Now, the student is just dealing with the numbers, without thinking of the situation. Finally, the fourth level would contain of the standard written algorithm for long division.

(3) The use of students own productions and constructions

Students should be asked to 'produce' more concrete things. De Lange(1995) stresses the fact that, by making 'free production', students are forced to reflect on the path they themselves have taken in their learning process and, at the same time, to anticipate its continuation. Free productions can form an essential part of assessment. For example, students may be asked to write an essay, to do an experiment, to collect data and draw conclusions, to design exercises that can be used in a test, or to design a test for other students in the classroom.

(4) The interactive character of the teaching process or interactivity

Interaction between students and between students and teachers is an essential part in RME (de Lange, 1996; Gravenmeijer, 1994). Explicit negotiation, intervention, discussion, cooperation, and evaluation are essential elements in a constructive learning process in which the student's informal methods are used as a lever to attain the formal ones. In this interactive instruction students are engaged in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting.

(5) The intertwining of various learning strands or units

In RME (de Lange, 1996; Gravenmeijer, 1994), the integration of mathematical strands or units is essential. It is often called the holistic approach, which incorporates applications, implies that learning strands can not be dealt with as separate entities; instead, an intertwining of learning strands is exploited in problem solving. One of the reasons is that applying mathematics is very difficult if mathematics is taught 'vertically', that is if various subjects are taught separately, neglecting the cross-connections. In applications one usually needs more than algebra alone or geometry alone.

4. How is the realistic approach related to the constructivist approach?

Due to many similarities with RME, the theory of constructivism in mathematics is included in this review. Some differences will be discussed as well. In general, constructivism means that programs start from the philosophy that give learners the freedom of their own construction or reconstruction. Three types of constructivism that are used in mathematics education are known as:

- radical constructivism: *knowledge can not simply be transferred ready-made from parent to child or from teacher to student but has to be actively built by each learner in his or her own mind* (Glasersfeld, 1992). Here, students usually do deal with meanings, and when instructional program fail to develop appropriate meanings, students create their own meanings. But Ernest (1991) argued this type of constructivism is lack of a social dimension in which the students learn dependently;
- social-constructivism: Ernest (1991) comes up with a new type of constructivism that is called social-constructivism which *views mathematics as a social construction which means that students can better construct their knowledge when it is embedded in a social process* (Ernest, 1991); and
- socio-constructivist: this type of social constructivism is developed only in mathematics education. The characteristics of this type are almost similar to the characteristics of RME such

as mathematics should be taught through problem solving, students should interact with teachers and other students as well, and students are stimulated solve problems based on their own strategies (Cobb, Yackel & Wood , 1992).

The fact that socio-constructivist is closely related to RME was stated by Gravenmeijer (1994) as well as de Lange (1996). There are two main similarities between RME and socio-constructivist mathematics education (de Lange, 1996). First, both the socio-contsructivist and realistic mathematics education are developed independently of constructivism. Second, in both approaches students are offered opportunities to share their experiences with others. In addition, de Lange (1996) stated that the compatibilities of socio-constructivist and RME are based on a large part or similar characterizations of mathemathics and mathematics learning. Those are: (1) both struggle with the idea that mathematics is a creative human activity; (2) that mathematical learning occurs as students develop effective ways to solve problems (Streefland, 1991; Treffers, 1987); and (3) both aim at mathematical actions that are transformed into mathematical objects (Freudenthal, 1991).

The main difference between RME and constructivism is that RME is only applied to mathematics education while constructivism is used in many subjects (de Lange, 1996). Moreover, Gravenmeijer (1994, p.81) pointed out that "the difference between socio-constructivist approach and realistic approach is that the former does not offer heuristics for developing instructional activities for students". In other words, in socio-constructivist approach, the teacher does not use heuristics, a method of solving problems by learning from past experience and investigating practical ways of finding a solution. In RME, it is known as guided reinvention.

5.How to design realistic mathematics education lesson?

Streefland (1991) developed realistic mathematics lessons (based on fractions in elementary school) using the three levels construction principle: (1) the local, or classroom level; (2) the global, or course level; and (3) the theoretical level.

(1) Classroom level

In this level, lessons are designed based on all the characteristics of RME and focus on the construction through horizontal mathematization (an example of the lesson is provided in the appendix). First, an open material is introduced into the learning situation and opportunity for carrying out free productions is provided. Then, characteristics of RME are applied to the lesson by: (1) situating the intended material *in reality*, which serves as *source* and as *area of application*, starting from meaningful contexts having the potential to produce mathematical material; involves; (2) *intertwining* with other strands; such as fractions and proportions; and (3) producing *tools* in the form of symbols, diagrams and situation or context models during the learning process through collective effort. Finally, (4) learning through constructions is carried out by arrangements of the students activities, so they can interact with each other, discuss, negotiate, and collaborate. And this is where the educational principle of *interaction* is applied. By this mean, the students contribution to their own learning path can be guaranteed. The students can be encouraged to follow this kind of constructional activity by giving them an assignment which leads to *free productions*.

(2) Course level

The material constructed at the classroom level is now used according to its mathematical and didactical essence in order to realize the general outline of the course. This means the measures taken to achieve contributions to the learning process at the local level must be continued at the general level.

(3) Theoretical level

All activities which took place in the both preceding levels such as design and development, didactical deliberation, and trying out in the classroom form the source of theoretical production, the generative material for this level. Constructing a theory in the form of a local theory for a specific area of learning. By using development research method, the local theory is revised and tested again in the other cyclic developments.

In order to design RME lessons, the components of a lesson plan will be identified and connected to realistic mathematics education. Those components are goals, content, methodology, and assessment.

(1) Goals

De Lange(1995) characterized three levels of goals in mathematics education: lower level, middle level, and higher order level. In the traditional program the goals were more or less clear. For example students should be able to solve a linear equation using a specific method. However, most of the goals of the traditional program are now classified as lower level goals that are based on formula skills, simple algorithms and definitions. In the realistic mathematics education goals are classified as 'middle' and 'higher' level goals. At the middle level, connections are made between the different tools of the lower level and concepts are integrated; it may not be clear in which strand we are operating, but simple problems have to be solved without unique strategies. This means that for both the teacher and the students the intended goals are not always immediately clear. Moreover, the new goals also emphasize the reasoning skills, communication and the development of critical attitude. These are popularly called 'high order' thinking skills. To conclude, in order to redesign a lesson based on the realistic approach it should contains these two types of goals.

(2) Materials

De Lange (1996) pointed out that materials are associated with real-life activities where domain-specific, situational knowledge and strategies are used within the context of the situation. A variety of contextual problems is integrated in the curriculum right from the start. In a general way , RME developers need to find contextual problems that allow for a wide variety of solution procedures, preferably those which, considered together, already indicate a possible learning process through a process of progressive mathematization.

(3) Activities

The role of the RME teacher in the classroom are (de Lange, 1996;Gravenmeijer, 1994): a facilitator, an organizer, a guide, and an evaluator. Based on the process of progressive mathematization, generally one can conclude that the role of teacher on the steps of the teaching-learning process based on realistic approach are:

- Give the students a contextual problem that relate to the topic as the starting point.
- During interaction activity, give the students a clue, for instance, by drawing a table on the board, guide the students individually or in a small group in case they need help;
- Stimulate the students to compare their solutions in a class discussion. The discussion refers to the interpretation of the situation sketched in the contextual problem and also focus on the adequacy and the efficiency of various solution procedures.
- Let the students find their own solution. It means the students is free to make discoveries at their own level, to build on their own experiential knowledge, and perform shortcuts at their own pace.
- Give another problem in the same context.

On the other hand, the role of students in RME are mostly they work individually or in a group, they should be more self-reliant, they can not turn to the teacher for validation of their answers or for directions for a standard solution procedure, and they are asked to produce free production or contribution.

(4) Assessment

In the Netherlands, development research on assessment based on the RME viewpoints carried out so far, has already produced some keys of how assessment can be improved, especially written assessment (Van den Heuvel-Panhuizen, 1996). In addition, doing assessment during the lesson, teachers may ask students to write an essay, to do experiment, collect data, and to design exercises that can be used in a test, or to design a test for other students in the classroom. Assessment can be continued by giving the students some problems as homework. But, in order to relate with the national standardized test, the assessment procedures should reflect the goals of the curriculum.

Regarding to assessment in RME, De Lange(1995) formulated the following five principles of assessment as a guide in doing assessment:

- The primary purpose of testing is to improve learning and teaching. It means assessment should measure the students during the teaching-learning process in addition to end of unit or course.
- Methods of assessment should enable the students to demonstrate what they know rather that what they do not know. It can be conducted by having the problems that have multiple solution with multiple strategies.
- Assessment should operationalize all of the goals mathematics education, lower, middle, and higher order thinking level.
- The quality of mathematics assessment is not determined by its accessibility to objective scoring. In this case, objective test and mechanical test should be reduced by providing the students with the tests in which we really can see whether they are understand the problems.
- The assessment tools should be practical, available to the applications in school cultures, and accessibility to outside resources.

In summary, figure 4 below shows how all the characteristics of RME are pictured in a model for designing RME lesson materials.

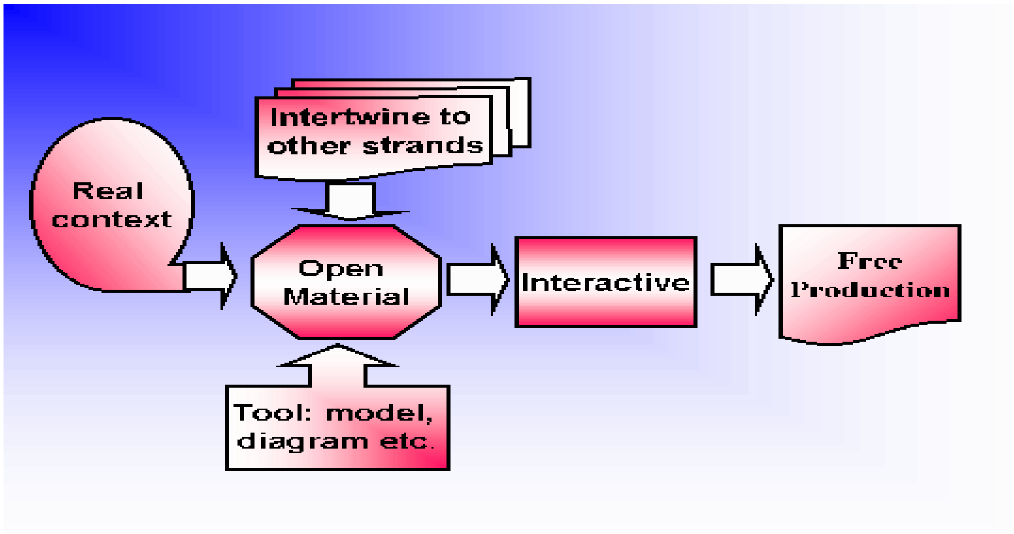


Figure 4. A model for designing RME Lesson Materials

The process of designing starting form an 'open material' that has opportunity for carrying out free productions. Then, characteristics of RME are applied to the lesson by:

- situating the intended material *in reality*, starting from meaningful contexts having the potential to produce mathematical material; involves;
- *intertwining* lines of learning with other strands; and
- producing *tools* in the form of symbols, diagrams and situation or context models during the learning process through collective effort;
- in the activity part of the lesson plan, the students are arranged so they can *interact* with each others, discussion, negotiations, and collaborations. In this situation they have opportunity to work with or doing mathematics, communicate about mathematics; and
- assessment materials should be developed in the form of open question which leads the students to *free productions*. The assessment should be given to the students either during or after the instruction process, or as the homework.

Finally, based on this model, an example of mathematics lesson plan is designed (see appendix).

6. Conclusion

In this concluding part, all of the questions are answered based on the explanation in each previous sections.

1. What is realistic mathematics education?

Realistic mathematics education is a theory in mathematics education that is originally developed in the Netherlands. It stresses the idea that mathematics is a human activity and mathematics must be connected to reality, real to the learner using real-world context as a source of concept development and as an area application, through process of *mathematization both horizontal and vertical*.

2. What are characteristics of realistic mathematics education?

Realistic mathematics education has five characteristics: (1) use real-life contexts as a starting point for learning; (2) use models as a bridge between abstract and real, that help students learn mathematics at different levels of abstractions; (3) use student's own production or strategy as a result of their doing mathematics; (4) interaction is essential for learning mathematics between teacher and students, students and students; and (5) connection to among strands, to other disciplines, and to meaningful problems in the real world.

3. How is realistic approach is related to the constructivist approach?

The realistic approach is similar to the socio-constructivist approach, except in socio-constructivist does not produce heuristics that can guide the development of instructional activities for students. In other words, in socio-constructivist approach, the teacher does not use heuristics, a method of solving problems by learning from past experience and investigating practical ways of finding a solution. In RME, it is known as guided reinvention.

4. How to design mathematics lessons based on the realistic approach?

The mathematics lessons that will be designed should represents all of the characteristics of RME especially in the material, activity, and assessment part of lesson plan. The way of embed these characteristics into the lesson plan components can be seen in the summary of previous section and an example of complete lesson plan that is designed based on the realistic approach can be seen in the appendix.

References:

Cobb, P.,E. Yackel, and T.Wood(1992). A Contructivist Alternative to the

Representational View of Mind in Mathematics education. *Journal for Research in*

Mathematics Education, 23(1), 2-33.

Ernest, P. (1991). *The Philosophy of Mathematics Education.* Hampshire: The Falmer

Press.

Freudenthal, H. (1991). *Revisiting Mathematics Education.* China Lectures. Dordrecht:

Kluwer Academic Publishers.

Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-ß

Press / Freudenthal Institute.

Lange, J. de (1996). Using and Applying Mathematics in Education. in: A.J. Bishop, et al. (eds). 1996. *International handbook of mathematics education, Part one. 49-97*. Kluwer academic publisher.

Lange, J. de (1995). Assessment: No Change without Problems, in: Romberg, T.A. (eds). (1995). *Reform in School Mathematics and Authentic Assessment*. New York, Sunny Press, 87-172.

Mullis, I.V.S., Martin, M.O., Beaton, A.E., Gonzalez, E.J., Kelly, D.L., and Smith, T.A. (1997). *Mathematics Achievement in the Primary School Years: IEAís Third International Mathematics and Science Study (TIMSS)*. Chestnut Hill, MA: Boston College.

Romberg, A. & Lange J. d (1998). *Mathematics in Context: Teacher Resource and Implementation Guide.* Britannica Mathematics system, USA.

Streefland, L. (1991). *Fractions in Realistic Mathematics Education. A Paradigm of Developmental Research.* Dordrecht: Kluwer Academic Publishers.

Streefland (ed.) (1991). *Realistic Mathematics Education in Primary School.* Utrecht: CD-b Press / Freudenthal Institute, Utrecht University.

Treffers, A. (1991). Realistic mathematics education in The Netherlands 1980-1990. In L. Streefland (ed.), *Realistic Mathematics Education in Primary School.* Utrecht: CD-b Press / Freudenthal Institute, Utrecht University.

Van den Heuvel-Panhuizen, M. (1998). *Realistic Mathematics Education, NORMA Lecture held in Kristiansand Norway*. Available on-line at http://www.fi.ruu.nl/en/rme

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht: CD-b Press / Freudenthal Institute, Utrecht University.

New Theory: Realistic Mathematics Education

Von Glasersfeld, E. (ed.). (1991). *Radical Constructivism in Mathematics Education*.

Kluwer, Academic Publisher, Dordrecht.

Appendix:**An example of a lesson material plan based on realistic approach**

Lesson: Introduction to the linear equation

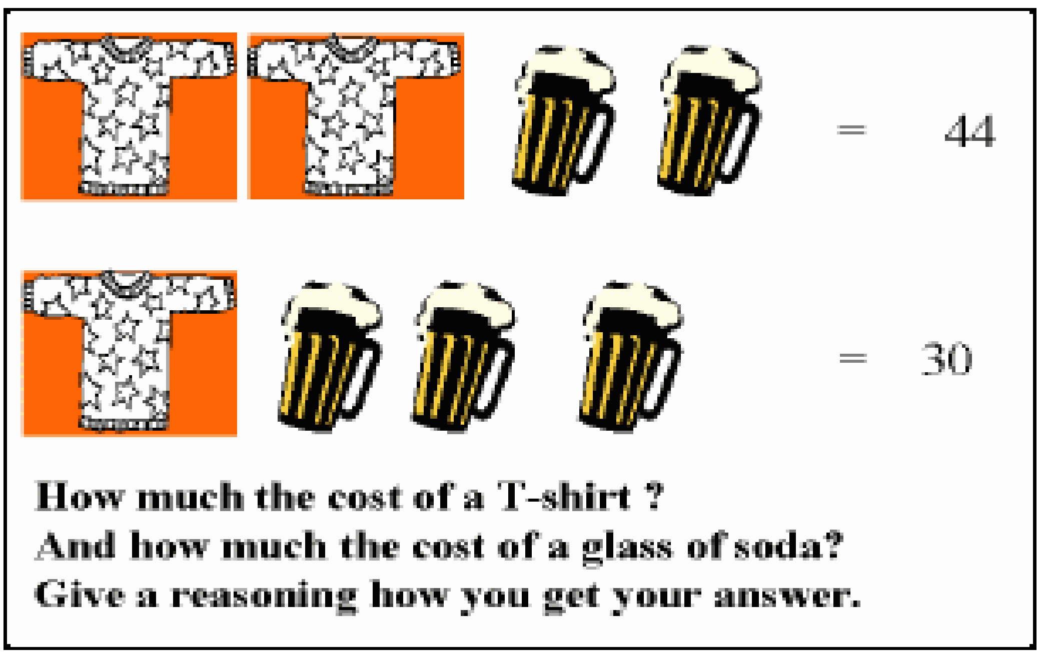
Time : (2 x 40 minutes)

Grade: Eight, Junior Secondary School students

Goals: After the students follow the lesson, they are able to:

- communication about mathematics during the discussion;
- learn to improve their reasoning skills by presenting the result of their work;
- learn how to improve their critical attitude by conflict to other students opinion;
- construct the relevant mathematics concept (in this case: linear equation) from a contextual problem, and
- produce a solution in their own strategies.

Material:



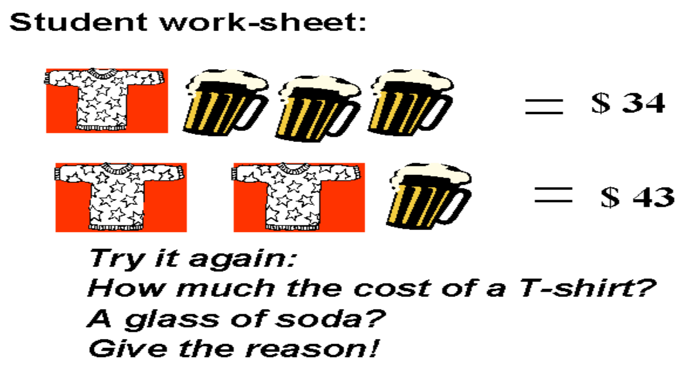
Activities:

- Give the students a contextual problem that related to the topic as the starting point
- By moving around find out which students or groups have the 'clever' or intended strategy. This information is important in discussing session.
- Stimulate the students to compare their solutions.
- Ask the student or the group of students to present their answer in front of the class.

- Guide the students in a class discussion

Assessment:

- during the instruction, give another problem in the same context (see figure below).



- As a home work, asking the students to write a short essay (on a piece of paper) about their experiences after they have learned the two lessons